Physical Problem Solving

Math Dance

with

Karl Schaffer
and
Erik Stern

based on the book
Math Dance with Dr. Schaffer and Mr. Stern
by Karl Schaffer, Erik Stern, and Scott Kim

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* Page numbers in parentheses refer to the book, "MathDance with Dr. Schaffer and Mr. Stern," provided to all workshop participants.

This workshop was developed in association with
The John F. Kennedy Center for the Performing Arts.

I. Introduction

A. Physical Problem Solving: Math Dance

Physical Problem Solving. *The process of solving physical problems challenges students to think and move clearly while engaging them creatively*. For example, one of the exercises in this workshop has trios of students creating a sequence of group shapes which exhibit rotational symmetry. Each member of the group must help to create and then remember the shapes, each must check if the shapes exhibit the symmetry by creating a three-dimensional picture of the group in their minds, each must decide how to maintain balance and coordinate the movements with other group members. *These physical problems lead to formal mathematical and choreographic discussions and also to performances.*

Math Dance. Dance and mathematics share many essential concerns: 1) they both deal with pattern recognition and manipulation, 2) they both involve defining a problem and seeking a solution, 3) they both begin with concrete problems and progress to abstract ideas, 4) they both involve aesthetics and are integrally connected to cultural values and biases, and 5) they both can make you sweat.

B. The Purpose of the Workshop

The purpose of this workshop is to give students a palpable experience to help them understand mathematical and choreographic concepts. Prior experience in mathematics and dance is not required. The workshop alternates between creative problem solving and reflection/discussion. Workshop participants will 1) solve problems physically in groups of two to four, 2) discuss problems in smaller groups and as a class, 3) will first explore creatively the subject matter and then formalize the concepts.

C. Rationale

"Learning Through Dance," a recent article published in the *American School Board Journal* (2000, Hanna), provides arguments for the role dance education plays in stimulating thinking, self-expression, and problem solving. The study of dance, according to the article, helped with perceptual awareness and encourages exploration of space, time, dynamics, gesture, phrasing, and motif.

The article cites a ten-year study of low-income youths. Regular study of the arts improved the youths' academic performance and increased their abilities in self-assessment.

The Development of Spatial Thinking in Schoolchildren, translated from Russian and published by the National Council for Teachers of Mathematics (1991, Yakimanskaya), is a scholarly work which demonstrates spatial thinking and its relationship to mathematics education.

Mathematicians view their work as having a creative component. Dancers/choreographers note the judicious thinking and attention to form which their art requires. Contrary to the perception of the two disciplines as opposites, they work well together.

D. Overview of the Workshop

E. Addressing State and National Standards

1. National Standards for Mathematics Education

Note: The foremost mathematics education organization in the United States, the National Council of Teachers of Mathematics (NCTM), published in 2000 Principles and Standards for School Mathematics (available at http://standards.nctm.org/). The Common Core State Standards for Mathematics (CCSSM), released in June 2010, have been adopted by 45 states. The CCSSM are similar in philosophy, though differing in specific grade level details, to the NCTM standards.

Grades K-2 (similar standards are found in grades 3-8)

All the mathdance activities meet the following standards:

Problem Solving

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.
- Monitor and reflect on the process of mathematical problem solving.

Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.

During the workshop the following standards are addressed:

- **Number:** Understand numbers, ways of representing numbers, relationships among numbers, and number systems. Count with understanding and recognize "how many" in sets of objects. Understand the effects of adding and subtracting whole numbers. Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. Understand meanings of operations and how they relate to one another. Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.

- **Algebra**: Understand patterns, relations and functions. Sort, classify, and order objects by size, number, and other properties. Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another. Analyze how both repeating and growing patterns are generated. Represent and analyze mathematical situations and structures using algebraic symbols. Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.

- **Representation:** Create and use representations to organize, record, and communicate mathematical ideas. Use representations to model and interpret physical, social, and mathematical phenomena.

- **Geometry:** Apply transformations and use symmetry to analyze mathematical situations. Recognize and apply slides (translations), flips (reflections), and turns (rotations). Recognize and create shapes that have symmetry. Use visualization, spatial reasoning, and geometric modeling to solve problems. Create mental images of geometric shapes using spatial memory and spatial visualization.

2. National Standards for Arts Education
Participating in this workshop's activities can contribute to fulfilling the following National Standards for Dance:

<u>**Grades K-4**</u>

- **Content Standard 1**: Identifying and demonstrating movement elements and skills in performing dance
  <u>Achievement Standard:</u> Students demonstrate kinesthetic awareness, concentration, and focus in performing movement skills.

- **Content Standard 2:** Understanding choreographic principles, processes, and structures
  <u>Achievement Standards</u>: Students create a dance phrase, accurately repeat it, and then vary it; Students demonstrate the ability to work effectively alone and with a partner.

- **Content Standard 3:** Understanding dance as a way to create and communicate meaning
  <u>Achievement Standards</u>: Students observe and discuss how dance is different from other forms of human movement (such as sports, everyday gestures); Students present their own dances to peers and discuss their meanings with competence and confidence.

- **Content Standard 4:** Applying and demonstrating critical and creative thinking skills in dance
  <u>Achievement Standard</u>: Students explore, discover, and realize multiple solutions to a given movement problem; choose their favorite solution, and discuss the reasons for the choice.

Grade 5-8

- **Content Standard 2:** Understanding choreographic principles, processes, and structures
  Achievement Standard: Students demonstrate the ability to work cooperatively in a small group during the choreographic process.

- **Content Standard 3:** Understanding dance as a way to create and communicate meaning
  Achievement Standard: Students effectively demonstrate the difference between pantomiming and abstracting a gesture.

- **Content Standard 4:** Applying and demonstrating critical and creative thinking skills in dance
  Achievement Standard: Students identify possible aesthetic criteria for evaluating dance (such as skill of performers, originality, visual and/or emotional impact, variety, and contact).

- **Content Standard 7:** Making connections between dance and other disciplines
  Achievement Standard: Students cite examples of concepts used in dance and another discipline outside the arts (such as balance, shape, and pattern).

F. Goals of the Workshop

Participants will . . .

1. ...experience physical problem solving and creating movements.

2. ...learn to draw on their own movement experiences when creating movement material.

3. ...discuss mathematical concepts and learn to use those concepts to create group sequences of movement.

4. ...reflect on the mathematical aspects of the movement sequences as they relate to counting, pattern, symmetry, and geometry.

5. ...reflect as a class on the aesthetics of the movement sequences and identify ways to further develop those sequences.

6. ...experience the joy of performing movement sequences for others.

7. ...appreciate the creativity and problem solving which dance requires.

8. ...find the mathematics and the dance in the world around them, in everyday things which they see and experience.

Dance Note: Everyday Movement
Everyday movements, like shaking hands, are a great way to involve non-dancers in doing and creating movement sequences.

Although many types of dance, such as classical ballet, take years of practice to master, others are built on everyday movements that anyone can do. For instance the musical performance Stomp creates dance out of everyday actions like sweeping the floor. Many hip-hop moves began as everyday gestures. All over the world folk dances are made out of the everyday movements of work and play.

In the world of modern dance, many choreographers during the 1960's used untrained dancers performing ordinary movements in their works. "Esplanade," by Paul Taylor, is built out of walking, running, and jumping movements. It takes concentration and focus to perform these dances well.

G. Is This For Everyone?

These activities are for everyone and do not require special training in dance or mathematics (although training or interest in these subjects can be helpful). The activities begin by addressing the most basic universal elements of dance and mathematics: recognizing and remembering patterns, walking, counting, moving, making shapes. The activities are designed to be flexible and can be extended to suit the level of the students.

For more information on how to prepare the classroom and students for these activities, please read pages 10-12 ("How to Use this Book") of the text Math Dance with Dr. Schaffer and Mr. Stern. This section of the book also covers basic concepts which underlie dance and mathematics.

H. Background

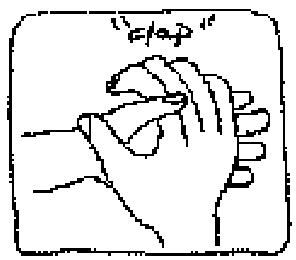
Karl Schaffer and Erik Stern had been choreographing works together for three years when they began to discuss the similarities between the processes which underlie mathematics and dance. The performance which resulted, "Dr. Schaffer and Mr. Stern, Two Guys Dancing About Math," premiered in 1990, has been performed over 500 times, and led to the creation of many other performances exploring the connections between mathematics and dance.

In 1993 Schaffer and Stern collaborated with Scott Kim, noted software designer and mathematician, on the performance "Dances for the Mind's Eye." As a natural outgrowth of their work on stage, Schaffer, Stern and Kim created workshops which allowed students to experience in the classroom the connections between dance and mathematics. The workshops have been given for ten years to tens of thousands of educators, artists, and students of all levels, from kindergarten through college. In addition, both the performances and workshops have been requested and delivered at scores of conferences, such as mathematics education conferences (including the National Council of Teachers of Mathematics national meeting) and arts education conferences (including the Dance and the Child International Festival). In 2001 the three collaborators collected their extensive classroom experience and the handouts they had been using and wrote the book "MathDance with Dr. Schaffer and Mr. Stern."

Aside from being a professional dancer, Karl Schaffer has a Ph.D. in Mathematics from University of California at Santa Cruz and teaches Mathematics at De Anza College in Cupertino, CA. Receiving an undergraduate degree in Biology, Erik Stern has an M.F.A. in Dance from the California Institute of the Arts and teaches Dance at Weber State University in Utah. Schaffer and Stern have been on the Touring Artists Roster of the Kennedy Center for the Performing Arts Partners in the Arts program since 2005, and travel extensively through that program doing teacher development work. They have also garnered five National Endowment for the Arts awards for their math/dance choreographic and educational work. Scott Kim is an internationally renowned puzzle-master and a mathematics educator and enthusiast who has a Ph.D. in Computer Science from Stanford University.

II. Math Dance

A. Clap Your Name (pages 23-30)

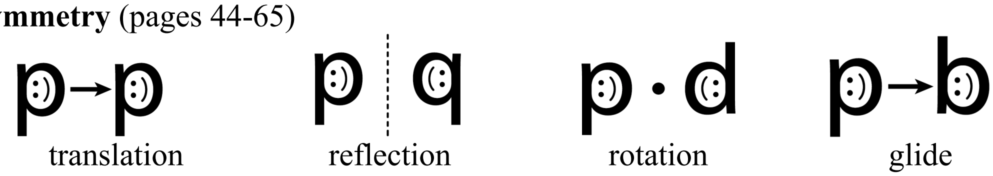


1. *In this activity participants create and perform rhythmic clapping patterns using their names, and then convert the patterns to movement sequences.* The activity helps develop number sense and addresses the concept of least common multiple. Participants look at basic choreographic principles of sequencing and transitions by creating and manipulating dance phrases. Participants also explore the relationship between pedestrian movement and dance. The connection between polyrhythm and least common multiple is also revealed.

2. Clap Your Name is an excellent "break the ice" activity and gently moves into movement. Participants pair up for the first exercise and can work at their desks. However, for the movement portion students will need to push desks to the side of the room.

3. Reflection. What is pattern? How does it relate to mathematics and movement? What adaptations would this activity require to best suit your students?

B. Symmetry (pages 44-65)



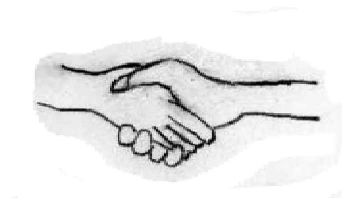
1. *In this activity, participants discover that there is more than one type of symmetry, then they create sequences of group shapes that exhibit the symmetries.* The different types of symmetry have long been applied to a variety of subjects such as mathematics, architecture, and visual arts. In dance, symmetry is a powerful guide to creating rich and challenging spatial relationships between dancers and also as a means of analyzing dance forms from around the world.

2. The room must have a large open space and be cleared of obstructions. Participants work in pairs and trios. Participants practice several forms of symmetry including translation, reflection, rotation, and glide. The study

of symmetry relates to dance, mathematics, chemistry, visual arts, crafts, architecture, and many other disciplines.

3. Reflection. Free-write on the questions: What disciplines besides mathematics and dance (both recreational and academic) involve symmetry? Which of these disciplines are taught in your class? How might movement activities facilitate an understanding of those other disciplines?

C. How Many Ways to Shake Hands? (pages 16-22)



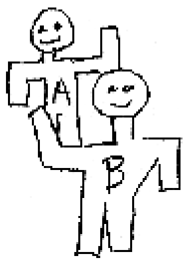
1. *In this activity participants create sequences of handshakes. Then they are asked to find the number of possible handshakes.* Resulting discussions lead to an understanding of counting, combinations, and problem definition. When working with a phrase of movement, manipulating that phrase by altering sequence and certain aspects of the movement is an essential part of tradition form known as theme and variation. On a broader level, defining the problem (in terms of subject matter, movement theme, visual theme, etc.) is a major aspect of the choreographic process. The question, "What belongs and what does not belong?" has far-reaching mathematical and artistic implications.

2. The room must have a large open space and be cleared of obstructions. Participants work in pairs and trios. This activity invariably leads to groups generating different answers, which then leads to discussions (sometimes heated!) of which count is correct. Aside from the mathematical and choreographic ideas which are inherent in this activity, it also challenges participants' ability to interpret rules and explain why an answer is consistent with that interpretation.

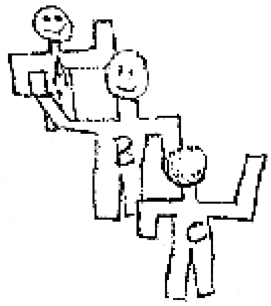
3. Reflection: What is the role of open-ended problems in education? When is it valuable and when is it not valuable? How does open-ended problem solving relate to the everyday world?

D. Twisted Addition (pages 66-76)

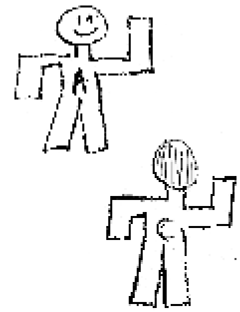
1. *In this activity participants investigate how symmetries combine to produce new symmetries.* In the process they glimpse the rich spatial structures that underlie all human movement, and observe how these structures may be examined more closely by translating them into symbolic form and back again into movement.



A and B in glide symmetry



B and C in rotational symmetry



A and C in mirror symmetry

2. The room must have a large open space and be cleared of obstructions. Participants work in trios and quartets. The participants use the Combining Symmetries black-line master to record their observations and continue to compose movement consciously utilizing the symmetries they are studying. Participants often remark on the surprising interplay between abstract concepts and movements of the body.

3. Reflection: How do we distinguish kinds of symmetries? What words do we use to describe them in dance and in other endeavors? How does symmetry arise in other ways than those we have explored?

III. Closure

A. Summary of Workshop Content
B. Benefits of this Work for Student Learning

IV. Supplementary Materials/References

A. Bibliography--see pages 127-130 in the text
B. Adding Music—see page 15 in the text
C. . Additional Activity: **Hand Figures** (pages 77-84)

1. In this activity participants make group shapes with their hands. Some of the shapes are of the group's invention and others are geometric shapes. Students engage in spatial thinking, an integral component of both dance and mathematics. Transitions between shapes develops coordination and choreographic decision-making.
2. Participants work individually and then in groups of two to four.
3. What is the role of the hands in the learning process?

D. Black-line Master — Twisted Addition

Combining Symmetries

Second symmetry: B to C

First symmetry: A to B

	T	G	M	R
T				
G				
M				
R				

T = translation (p→p) G = glide (p→b) M = mirror (p→q) R = rotation (p→d)

Quarter Turns

Second turn

First turn

	0	1	2	3
0				
1				
2				
3				

0 = no turn (p→p) 1 = quarter turn (p→ᓄ) 2 = half turn (p→d) 3 = three quarters turn (p→ᓀ)

Placing Pre-mathematical Thinking In Context

"Vision, spatial sense, kinesthetic (motion) sense. People have very powerful facilities for taking in information visually or kinesthetically, and thinking with their spatial sense. On the other hand, they do not have a very good built-in facility for inverse vision, that is, turning an internal spatial understanding back into a two-dimensional image. Consequently, mathematicians usually have fewer and poorer figures in their papers and books than in their heads.

"An interesting phenomenon in spatial thinking is that scale makes a big difference. We can think about little objects in our hands, or we can think of bigger human-sized structures that we scan, or we can think of spatial structures that encompass us and that we move around in. We tend to think more effectively with spatial imagery on a larger scale: it's as if our brains take larger things more seriously and can devote more resources to them."

"One-on-one, people use wide channels of communication that go far beyond formal mathematical language. They use gestures, they draw pictures and diagrams, they make sound effects and use body language. Communication is more likely to be two-way, so that people can concentrate on what needs the most attention. In talks, people are more inhibited and more formal…in papers people are still more formal. Writers translate their ideas into symbols and logic. And readers try to translate back… Mathematics in some sense has a common language: a language of symbols, technical definitions, computations, and logic. This language efficiently conveys some, but not all, modes of mathematical thinking."

William P Thurston, "On Proof and Progress in Mathematics," in *18 Unconventional Essays on the Nature of Mathematics*, ed. By Reuben Hersh, Springer, New York. 2006.

Pre-mathematics: Educators Versus Mathematicians

"One of the cultural divides between mathematics educators and mathematicians is, what stage in the life cycle of a mathematical idea do they focus on? By and large, educators are interested in the formation of the intuitive, pre-formal idea in mathematics learning. Mathematicians are interested in formal statement and proof--the finished mathematical idea. But then neither one is focusing on the most significant puzzle of mathematical learning: the process of metamorphosis! We need a theory of change from pre-mathematical to mathematical thinking. To that end, we would like to propose here that *mathematical discourse* plays an important role in the transition from intuitive mathematical thinking into critical thinking. "

"Mathematical concepts are shared concepts. How is common understanding achieved? Through mathematical discourse. What characterizes mathematical discourse? Repeated refinement of ideas, struggle with meaning, and the continual requirement to justify one's ideas. Mathematical discourse seeks to broaden, deepen, and clarify the objects and structures of mathematics. It isn't limited to verbal discussion between two people; it includes reading and writing texts, listening to and giving lectures."

Kristin Umland and Reuben Hersh. "Mathematical discourse: the link from pre-mathematical to fully mathematical thinking." Philosophy of Mathematics Education Journal No. 19 (December 2006). Accessed 8/10/07 at http://www.people.ex.ac.uk/PErnest/pome19/index.htm

The Kennedy Center for the Performing Arts definition of Arts Integration

Arts Integration is an approach to teaching

in which students construct and demonstrate understanding

through an art form.

Students engage in a creative process

which connects an art form and another subject area

and meets evolving objectives in both.