Dancing Permutations

Exploring Sequences and Combinations in Mathematics and Dance

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These pages are excerpts – contact us for complete workshop materials

OVERVIEW

Combinations and permutations in essence deal with the ways in which items can be combined and rearranged. They are used all the time in the arts and in mathematics. These concepts come into play when people assign and manipulate phone numbers, create theme and variation, use library reference systems, make orchestral scores, study DNA sequences in genetics, and in many other areas. In this workshop, students create dance phrases using four movements. They explore some of the possible rearrangements of four things, and address the mathematics and dance principles involved. How many ways are there or order four movements? The students attempt to answer this question and learn about the factorial operation. In mathematical parlance, order matters in a permutation; in a combination, order does not.

Grades:	3-8
Time:	60-90 minutes (but can be separated and broken down into shorter sections)
Concepts:	Math: counting, sequence, order, addition, multiplication, factorial, combination, permutation, sample space, probability Dance: phrase, transition, level, beginning, ending, unison, rehearsal, performing, variation, sequence
Groups of:	Individual, then 3 or 4
Space:	A large open room, such as a gym or multi-purpose room
Materials:	Board to write on. Paper and pencil (not required)
Related activities:	Chapter 1 *How Many Ways to Shake Hands?,* Chapter 2 *Clap Your Name* ,and Chapter 3 *Heads or Tails?* from our book *Math Dance*, available on Amazon. All involve counting problems

1. Make a Move (10-20 minutes)

- After a brief warm-up, have each student make up a "Move" or signature movement. *(For Tips on Leading Movement look at page 34. For more ideas on warmups see pages 23 and 24)* It should be concise. (NOTE: for a detailed description of the warm-up, see Support Materials) The Move can be fluid and dance-like, or athletic; it can be quirky or draw on vocabulary from other movement forms. It helps to give two examples (try to make them contrast).
- Once the class seems ready with their Moves, divide them in half. Each half demonstrates their "Move" at the same time while the other half watches. Notice if there are similar movements. If so, suggest ways to develop the Moves. Possible ways to develop: play with levels, alter the speed, alter the size, add arms, add legs, make it travel, have it express the way they feel that day, etc. Next, ask the students to clarify exactly how the Move begins and ends (What position are they in? Where is the weight? How does the Move get started?) Have the halves perform again, but this time do the Moves two or three times in a row. Make sure they complete a Move before beginning it again.
- Arrange the class in groups of three or four. Go around the room and have each group demonstrate

their individual Moves at the same time. This is a way for students to see all the Moves in the class and get a feel for them. As the demonstration is going on, look for Moves that are clear, interesting or original. Attributes of strong Moves include:

Size, it covers space or is athletic
Flowing, it has a fullness and connection
Quirky, it expresses individuality
Expressiveness, it conveys an idea or feeling
Clarity, the movement is specific and consistent

2. Create a Phrase (10-20 minutes)

- Choose four Moves from the class to create a phrase. Look for contrasting Moves. For example, if one student has a Move with a good jump, for the next Move look for one that is smooth or uses the lower space. Choose them one at a time, having the participants learn each Move. The teacher can choose, have students volunteer their Moves, let it be a group decision, or have one student choose. When the class is learning a Move, the student whose Move was chosen comes to the front of the class and demonstrates. The class should learn the Move by following the leader and repeating it several times. Avoid discussing the Move at first (often, discussing a Move changes it). Once the class has performed it a few times, ask for questions in case clarifications are needed.
- Repeat the process for the next three moves, adding each Move to the sequence. For example, after Move 2 has been learned, have the class perform Move 1 and 2 in sequence. It's important to have a leader for everyone to follow. Decide as a class how to transition from Move 1 to 2. It doesn't have to be tricky, just clear. For example, if Move 1 ends in a crouching position and Move 2 begins balancing on a straight leg, decide how to get from the crouch to the one-leg balance. Ask for suggestions: does the class want to jump, or simply stand on the balance leg, or turn around?
- Once the four Moves are known, write the Moves on the board, including a short description:

Jesse's Move: Sweeping Down
Alehandra's Move: Stork Gone Mad
Pete's Move: Water
Teisha's Move: The Washing Machine

- Practice the dance phrase as a class. In the halves used for the first exercise, have half perform the sequence while the other half observes. Ask the class, "What did you like about the sequence?" "What could be improved?" If needed, practice the phrase again.
- Ask the class for impressions of the sequence (or dance phrase, routine, combination . . .) Is it a random sequence of movements, or does it seem to be suggesting some kind of interpretation, for example: cheerleading, an emotional outburst, a greeting, a folk dance. If the dance phrase was developed in a manner consistent with the four movements so far, what kind of dance would they seem to be creating? Can they offer simple suggestions about how to develop it further?

3. Make Your Own Dance Phrase (10-20 minutes)

- Return to the groups of three or four used earlier in the class (when they demonstrated Individual Moves together). Each group works on this problem: <u>As a group, find a different order for the four Moves.</u> The new order can be chosen, for example, by rearranging numbers or Move names, or by dancing a Move and looking for the next Move that fits best. For example, one group might choose the order numerically, 3-4-2-1. Another group might put the names in an order they like: Stork Gone Mad to Washing Machine to Water to Sweeping Down, and so on. We suggest you don't allow the order 4-3-2-1 (it's popular choice and the goal is to get as many *different* orders as possible).

- Once the group has decided on the order, practice so the group can perform the new order.

Note: Look carefully at transitions - how one gets from one Move to the next. The group gets to choose its transitions. A transition can be simple and direct, can be predicable or surprising. The important thing is to try not to change the Moves, and to commit physically. The four Moves need to stay recognizable, but the transitions can and should vary. Allow time to practice, making sure each group can perform the new sequence without stopping, interrupting the flow, or talking. Have each group demonstrate. This is best done to music CD (music without words is best), or to live drum accompaniment. If time allows, practice the dance with music. Set the order in which the groups will perform, and show.

4. Reflection on Dance and Mathematics (5-10 minutes)
After the performance, ask students:

- What did you notice about the dances?
- Could you tell which order each group performed?
- What was hard about the assignment?
- What was fun about it?
- Which phrases seemed to catch your attention? Why?

In mathematics, we often give each permutation giving equal weight or importance. For example, we might try to count all the permutations that are possible, in which case each permutation "counts" the same. However, in dance (other art forms) we often play with different arrangements of movements and find that some of the permutations work better than others. Different art forms – and different artists – accomplish this experimentation in a wide variety of ways. Playing with the elements of an art form is central to the creative process. Often the result of this process is that several of the permutations are used in the finished work, and this is often central to the use of what is called "theme and variation."

Transition can be tricky. Ask students: What did you learn about these four movements and the transitions between them, in carrying out this study? What did you learn about experimenting with movement sequences? Would it be interesting to see a dance in which a number of the permutations are performed? Why or why not?

Then ask: How many possible sequences are there for these four dance Moves, assuming each Move has to be used once and only once? Let trios and quartets talk it over. Write the guesses down on the board. If time allows, try writing all the sequences down. (They may want to abbreviate the Moves in the list, for example by using 1,2,3, and 4.) If they do this, have them count them all. Answer is 24.

5. How Many Permutations Are There? (10-30 minutes)
In mathematics, we use the term **permutation** to mean an arrangement of objects in which order matters. A **combination**, on the other hand, is a selection or subset of some of the objects, and in this case order is not important. However, in dance we often use the term **combination** to mean a sequence of movements meant to be performed in a specified order. For clarity, in this handout we will use the terms *permutation*, *order*, *arrangement*, or *sequence* if order matters, and *combination* if it does not.

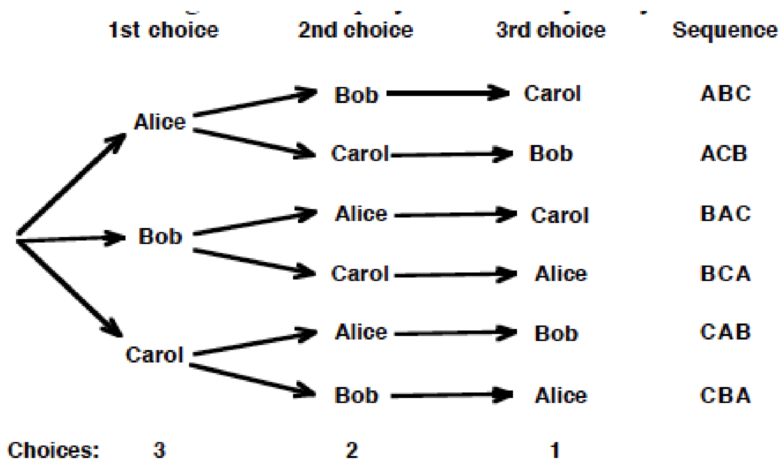
Write down on a board all the orders the groups performed (e.g. 1-2-3-4, 3-2-4-1-, etc.) Ask them: How many orders, or permutations, are there? Take guesses and explanations from the class, but don't worry about getting the right answer.

If the class needs to see a simpler version of the problem, ask for a volunteer to stand at the front of the room. Ask, “How many orders are there for one person to stand in a line?” There is only one way, since there is no one else to stand in front or in back of him or her!

Ask for a second volunteer to join the first, and then ask, “How many orders are there if two people stand in line?” In this case there are two, and you can ask the volunteers to demonstrate.

Then have a third volunteer join the first two, and ask, “How many orders are there if three people stand in a line?” In this case have the volunteers try to find them all. The non-volunteers can be encouraged to help. There are six possible orders.

For example, if the three people are Alice, Bob, and Carol, then the diagram shows a “tree” diagram that displays the six ways they can stand in line:



The tree diagram helps us see that there are three groups of two groups of one, or 3 X 2 X 1 = 6 possible sequences.

Another option is to take three small objects that are easily held in your hand, such as a pen, keys, a stapler, etc. Hold up one object and ask: How many orders can this be in? The answer is one. How up two objects and ask: How many orders can these be arranged in? The answer is two. Demonstrate. Hold up three objects and ask: How many orders can these be arranged in? The answer is six. See if the class can identify those six orders.)

Ask: Are there patterns we are seeing as we increase the number of people? Students might notice:

• The number of permutations increases quickly with the number of people.
• It begins to get very hard to remember all the sequences.
• As people are added some participants may notice that we seem to multiply the previous number of permutations to get the next number of permutations.

Tell the class that they will try to figure out more about this later. But for now the class will look at permutations from the perspective of dance phrases.

The Multiplication Principle.
If there is time, you may want to help the participants organize all the sequences. One way to do this is to ask the following questions:

"How many ways are there to choose the first movement?" The answer is four.

"After choosing the first Move, how many ways are there to then choose the second one?" Since one movement has already been used, there are three left, so there are three choices for the second movement. We thus need to multiply 4 by 3 to see how many we have so far. This is called the **Multiplication Principle**. You might want to illustrate this with a tree diagram.

(NOTE: if time allows, we like to ask, "What do you do with the 4 and the 3?" Adults as well as children often have a hard time explaining why multiplication is necessary rather than addition. This conversation is the essence of mathematical thinking.)

There are thus 4 X 3 =12 different arrangements so far. Continuing in this way, you will find that there are 4 X 3 X 2 = 24 arrangements of three of the four Moves, and 4 X 3 X 2 X 1 = 24 for all four Moves.

A key characteristic of art forms that separates them from the abstractions of mathematics is that usually not all such permutations are of equal interest. However, often the ones that are of interest exhibit further patterns. For example, in square dancing, as performed traditionally, not all dancers partner with each other, only men and women, so that perhaps only half of the possible permutations will be used in the dance!
Write on the board:

1 object . . . 1 permutation
2 objects . . . 2 permutations
3 objects . . . 6 permutations
4 objects . . . 24 permutations

Ask: Is there a pattern in this list? Answer: Multiply the sequence of numbers together to get the answer.

Two objects: multiply 1 x 2 = 2 permutations
Three objects: multiply 1 x 2 x 3 = 6 permutations
Four objects: multiply 1 x 2 x 3 x 4 = 24 permutations
Five objects: multiply 1 x 2 x 3 x 4 x 5 = 120 permutations

This calculation is called "factorial." Below are two common definitions:
$n! = 1\cdot 2\cdot 3\cdot \ldots \cdot (n-2)\cdot(n-1)\cdot n$ is the product of all positive integers less than or equal to n.
or
n! is defined "recursively" by the following
(a) $0! = 1$
(b) $n! = n\cdot(n-1)!$

(This is the definition used in computer programming; it is efficient since the program "calls" on itself repeatedly until it reaches 0! Odd though it may seem, 0! is defined as 1 in order to be consistent with rules for using factorial.)

Background

Karl Schaffer and Erik Stern had been choreographing works together for three years when they began to discuss the similarities between the processes that underlie mathematics and dance. The performance that resulted, *Dr. Schaffer and Mr. Stern, Two Guys Dancing About Math*, premiered in 1990, was performed over 500 times throughout North America, and led to the creation of many other performances and workshops exploring the connections between mathematics and dance. Their workshops such as "Dancing Permutations" have been given for over thirty years to tens of thousands of dancers, educators, scientists and mathematicians, and students at all levels, from kindergarten through college. Schaffer and Stern are on the Kennedy Center Partners in Education Teaching Artist Roster and have presented their work worldwide Aside from being a professional dancer and choreographer, Karl Schaffer has a Ph.D. in Mathematics from University of California at Santa Cruz and is Prof. Emeritus of Mathematics at De Anza College in Cupertino, CA. Receiving an undergraduate degree in Biology, Erik Stern has an M.F.A. in Dance from the California Institute of the Arts and is a Professor of Dance at Weber State University in Utah.

